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The gravity dual of softly broken $\mathcal{N} = 1$ super Yang-Mills

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Abstract: Starting from the Maldacena-Nunez supergravity dual of $\mathcal{N} = 1$ super Yang-Mills theory we study the inclusion of a supersymmetry breaking gaugino mass term. We consider a class of non-supersymmetric deformations of the MN solutions which have been recently proposed in the literature. We show that they can be interpreted as corresponding to the inclusion of both a mass and a condensate. We calculate the vacuum energy of the supergravity solutions showing that the N_c-fold vacuum degeneracy of the $\mathcal{N} = 1$ theory is lifted by the inclusion of a mass term.

Keywords: AdS/CFT and dS/CFT Correspondence, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

The AdS/CFT correspondence [1] provides the prototype example of a duality between a gauge theory ($\mathcal{N}=4$ Super Yang-Mills (SYM) in four dimensions) and a string theory (type IIB on the background $AdS_5 \times S^5$). There has been great interest in trying to extend this duality to other less supersymmetric gauge theories. Much attention has focused on theories with less, but non zero, supersymmetry, where gauge theory is more clearly understood. Supergravity backgrounds have been found both by the deformation [2] of the $\mathcal{N}=4$ theory by relevant perturbations and from consideration of the near horizon limit of more complicated D-brane constructions [3]. For example, the $\mathcal{N}=2^*$ theory has been obtained by including masses that break $\mathcal{N}=4$ to $\mathcal{N}=2$ in the infrared [4], while the pure $\mathcal{N}=2$ theory has been realized from D5-branes wrapped on a two cycle [5] or from branes at an orbifold singularity [6]. The $\mathcal{N}=1^*$ theory has been studied both as a deformation [7] and by using fractional branes [8]. In this paper we shall focus on the dual of $\mathcal{N}=1$ SYM provided by Maldacena and Nunez resulting from D5-branes wrapped on a two cycle [9], based on the solution in [10]. In each of these cases it has been understood how the gravity dual reproduces the perturbative running of the gauge theory though non-perturbative aspects of the theory are more hit and miss depending on what phenomena survive the large-N_c limit. For example the Maldacena-Nunez $\mathcal{N}=1$ solution includes a gaugino condensate whilst in the $\mathcal{N}=2$ theory instanton effects are squeezed into an enhançon singularity [11] of the supergravity background.

Dualities with string theories appear to be a robust phenomena in gauge theory and it is therefore interesting to now try to extend our understanding beyond supersymmetric theories. One might hope eventually to have a description of strongly coupled Yang Mills theory or even QCD. A few steps have already been taken in this direction, including studying non-supersymmetric deformations of the $\mathcal{N}=4$ theory with 5-dimensional supergravity [12], type-0 theories [13] and finite temperature theories [14].[1] In the interesting

[1]See [15] for the study of a metastable $N=0$ solution.

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case of zero temperature, all these solutions are plagued by singularities. More recently the authors of [16] studied the inclusion of supersymmetry breaking scalar operators in the Maldacena-Nunez $\mathcal{N}=1$ theory, which results in a regular background. In this paper, we wish to study the inclusion of a gaugino mass term rather than scalar operators. Again a set of IR regular solutions can be found. The gaugino mass term has the interesting property of breaking the $U(1)_R$ symmetry of the $\mathcal{N}=1$ theory. The result of this, as we show by calculating the vacuum energy of the appropriate supergravity backgrounds, is to lift the N_c-fold vacuum degeneracy of SU(N_c) $\mathcal{N}=1$ SYM. Much of the computational technology we shall use has already been studied in [17] where non-BPS versions of the Maldacena-Nunez solution were exhaustively presented. The purpose of [17] was different from ours, the authors being interested in finite energy excitations of the MN solution, with zero or finite temperature. We show that a class of solutions in [17] can be interpreted as a gravity dual of softly broken $\mathcal{N}=1$ SYM. In order to be self-contained, we will review most of the computation in [17]. We will then make contact between these gravity solutions and the physics of the dual gauge theory.

Let us review the field theory expectations when a gaugino mass is introduced into $\mathcal{N}=1$ SU(N_c) SYM. The $\mathcal{N}=1$ theory is known to have a mass gap due to the formation of a gaugino condensate and the vacuum is therefore described by the holomorphic superpotential

$$W=\Lambda^3 e^{in2\pi/N_c}\,, \qquad n=0,\ldots,N_c-1\,, \tag{1.1}$$

where Λ may be written in terms of the bare coupling $\tau=\frac{\theta}{2\pi}+i\frac{4\pi}{g^2}$ at some UV scale as

$$\Lambda=\Lambda_{UV}e^{2\pi i\tau/3N_c}\,. \tag{1.2}$$

The N_c-fold degeneracy of the vacuum corresponding to the N_c choices of phase in (1.1) is related to the anomalous breaking of the $U(1)_R$ symmetry of the theory to Z_{2N_c} by instanton effects (the Z_2 symmetry on the gaugino $\lambda\to-\lambda$ is left unbroken by the bifermion condensate).

Soft breaking terms may be introduced into supersymmetric theories by allowing the parameters of the theory to have non-zero F-components [18]. If τ has a non-zero F-component, f_τ, then in the bare lagrangian a gaugino mass is introduced. If the mass is small relative to Λ the supersymmetry breaking term will act as a perturbation to the stable N_c vacua and the resulting theory will still be described by (1.1). Accounting for f_τ it can be seen that the vacuum energy is no longer zero but, at leading order in the mass, it is given by (for $\theta=0$ and f_τ real) [19]

$$\Delta V=-32\pi^2 m_\lambda\Lambda^3\cos\left[\frac{2\pi n}{N}\right]. \tag{1.3}$$

The plane of the vacua is tilted and there is a single unique vacuum ($n=0$).

This is the property of the softly broken theory we uncover in the supergravity dual below. In the next section we will review the Maldacena-Nunez solution and identify the field corresponding to the gaugino condensate. In section 3 we show how a more general solution of the second order supergravity equations allows the inclusion of a mass term for

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the gaugino. Section 4 describes the determination of the vacuum energy of the spacetimes describing the perturbed vacua of the $\mathcal{N} = 1$ theory and reproduction of the field theory result eq. (1.3).

2. Gravity dual of $\mathcal{N} = 1$ SYM

Consider a wrapped five-brane with world-volume $R^4 \times S^2$. This general setting can be easily adapted to describe both $\mathcal{N} = 2$, $\mathcal{N} = 1$ and $\mathcal{N} = 0$ theories.

In the case of N_c flat NS5-branes the world-volume theory is a little string theory which reduces in the IR to $\mathcal{N} = 1$ six-dimensional SYM theory. The theory contains 4 scalars transforming in the 4 of the $\mathrm{SO}(4)_R$ R-symmetry group, and two symplectic Majorana fermions transforming in the $(4, 2) + (4', 2')$ of $\mathrm{SO}(5, 1) \times \mathrm{SO}(4)$. Wrapping the brane on an S^2 one obtains 4-dimensional gauge theories with coupling inversely proportional to the volume of the S^2.[2] Since there are no covariantly constant spinors on S^2, supersymmetry is generally broken by the compactification. In order to preserve some supersymmetry the theory has to be twisted, namely the spin connection on S^2 has to be identified with a background $U(1)_R$ field in the $\mathrm{SO}(4)_R$ R-symmetry group [20, 9]. This can be easily seen from the supersymmetry variation of a fermion

$$\delta\Psi \sim D_\mu \epsilon = (\partial_\mu + \omega_\mu^{\nu\rho}\gamma^{\nu\rho} - A_\mu)\epsilon\,. \tag{2.1}$$

The choice of the U(1) in $\mathrm{SO}(4)_R$ determines the amount of the supersymmetry left. For $\mathcal{N} = 1$ supersymmetry the relevant twisting $U(1)_R$ is the abelian subgroup of $\mathrm{SU}(2)_R$ in the decomposition $\mathrm{SO}(4) \to \mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$.[3]

As is standard in the AdS/CFT correspondence, the SO(4) gauge fields correspond to the isometries of the 3-sphere and are dual to the R-symmetry group. In the 7-dimensional field theory we are including a source term with the symmetry properties of a $U(1)_R$ gauge field.

The massless four-dimensional fields in the $\mathcal{N} = 1$ theory are the gluons and the gluino λ. The latter is the only component of the S^2 reduction of the five-brane fermionic field Ψ satisfying the twist condition: $(\omega_\mu^{\nu\rho}\gamma^{\nu\rho} - A_\mu)\epsilon = 0$. All other scalars and fermions from the reduction of the six-dimensional theory acquire mass due to the twist.

The dual supergravity background has been constructed, as usual, by first reducing to a lower-dimensional gauge supergravity and then by lifting the solution to 10 dimensions. In this case, the relevant theory is 7-dimensional SO(4) gauged supergravity, which corresponds to the truncation of the type-I sector of type IIB on the 3-sphere transverse to the NS5-brane. This is a consistent choice since the NS5-branes only couple to the NS sector of type-IIB SUGRA [9]. In fact the solution of [9] is the lift to 7 dimensions of a

[2] The presence of the S^2 naturally implies the existence of Kaluza Klein modes in the theory. Since the theory is at large $g_{\mathrm{YM}}^2 N_c$ these massive modes can not be considered decoupled and in this sense the dual gauge theories are not the pure 4-dimensional theories one might have hoped for. Nevertheless we hope they lie in the same universality class.

[3] Solutions with $\mathcal{N} = 2$ supersymmetry have been discussed in [5]. In that case, the U(1) field is a combination of the two abelian subgroups $U(1)_L + U(1)_R$.

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non-abelian monopole solution of 4-dimensional SU(2) gauged supergravity found in [10]. In a 7-dimensional string frame the solution describing wrapped NS5-branes consists of the the metric

$$ds_7^2 = dx_4^2 + \frac{N}{4}\left[dr^2 + e^{2g}(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{2.2}$$

the $SU(2)_R$ gauge fields

$$A = \frac{1}{2}\left[\sigma^3\cos\theta d\phi + \frac{a}{2}(\sigma^1 + i\sigma^2)(d\theta - i\sin\theta d\phi) + \text{c.c}\right], \tag{2.3}$$

and the dilaton

$$e^{2\Phi_{NS}-2\Phi_{NS0}} = \frac{e^g}{\sinh r}. \tag{2.4}$$

The functions

$$\begin{aligned} a &= \frac{r}{\sinh r}e^{i\chi}, \\ e^{2g} &= 2r\coth r - |a|^2 - 1 \end{aligned} \tag{2.5}$$

have been obtained from the supersymmetry variations in the 7-dimensional theory [9]. Here a is a complex field whilst g and Φ_{NS} are real. The solution has non-vanishing non-abelian gauge fields, which go to zero in the UV and reduce to a pure gauge in the IR. Their inclusion is required in order to have a regular solution and it breaks the $U(1)_R$ symmetry of the theory to Z_2 in the IR. To describe a decoupled four-dimensional theory we need to go to a D5-brane description [9]. The supergravity solution for a wrapped D5 is obtained by S-duality from the NS5 solution. In the next sections we will always consider wrapped D5's.

The supergravity fields entering the Maldacena-Nunez solution are dual to four-dimensional composite operators. In particular the field a, which is necessary in order to have a regular solution, is dual to the gluino condensate. This follows from the gauge fields in (2.3) and the S^2 reduction of the six-dimensional lagrangian term [21]

$$\bar\Psi A_\mu\gamma^\mu\Psi \longrightarrow a\bar\lambda\lambda. \tag{2.6}$$

As usual in the AdS/CFT correspondence, the UV gauge theory is determined by the large-r behaviour of the solution. At large r we find

$$a(r) \sim Kre^{-r}, \tag{2.7}$$

where $K = 2e^{i\chi}$ should be interpreted as (the complex conjugate of) the condensate [21]. Note that in the equation above we consider the full complex a field since it allows us to describe the gaugino condensate including its phase, χ. In this formula, the condensate has a free phase. The anomaly restricts χ to discrete values $2\pi n/N_c$ corresponding to the N_c vacua. The anomaly has been identified in the behaviour of the antisymmetric field B_{NS} [22] and in the contribution of world-sheet instantons [9] in the 10d lift of the solution. As shown in [9], only these discrete values of χ give rise to fully consistent 10-dimensional solutions. By considering the IR part of the metric, one can see that each of them only preserves a Z_2 symmetry, consistent with QFT expectations.

3. A soft breaking mass

To introduce a mass term into the $\mathcal{N}=1$ solution we note that a gaugino mass and a gaugino condensate share the same symmetry properties (up to conjugation) and should therefore both be described by a more general solution for a. Since a mass term for the gaugino breaks supersymmetry, on the supergravity side we can no longer look at the variations of the fermions, but we need to solve the second order supergravity equations of motion. Fortunately, much of the work has been done, for a different purpose, in [17]. In particular, what we need is the 1-dimensional effective Lagrangian describing the fields we are interested in [17]

$$\mathcal{L} = e^{2s}\left(s'^2 - \frac{e^{-2g}}{2}|a'|^2 - \frac{g'^2}{2} - \frac{1}{4}\left[e^{-2g}(|a|^2-1)^2 - 2e^{-2g} - 1\right]\right). \tag{3.1}$$

Here $s = \Phi + g$ and $\Phi = -\Phi_{NS}$ is the dilaton in the D5 description. The second order equations of motion are then given by

$$\begin{aligned}
\left[e^{2s-2g}a'\right]' &= e^{2s-4g}(|a|^2-1)\ a\,,\\
\left[e^{2s}(s')\right]' &= \frac{1}{2}e^{2s}\left[-e^{-4g}(|a|^2-1)^2 + 2e^{-2g} + 1\right],\\
\left[e^{2s}g'\right]' &= e^{2s}\left[-e^{-4g}(|a|^2-1)^2 + e^{-2g}\right] - e^{2s-2g}\ |a'|^2\,.
\end{aligned} \tag{3.2}$$

These equations admit the supersymmetric solution, which corresponds to the Maldacena-Nunez background. The most general solution to the second order equations of motion for a, which we present in full below, admit at large r, a non-normalizable asymptotic solution $a_1 \sim 1/\sqrt{r}$ and a normalizable one $a_2 \sim re^{-r}$. As we have seen above, a background where only the normalizable solution is turned on is associated with a vacuum of the field theory with a VEV for the corresponding operator. The non-normalizable solution a_2 changes the UV behaviour of the solution and it is therefore associated with a deformation of the theory where the gaugino has a mass.[4]

In general, therefore, the solutions of the second order equations, with two complex free parameters associated with a, will describe both the gaugino condensate and a mass term for the gaugino. These solutions will generically break supersymmetry. The full asymptotic behaviour for large r reads

$$\begin{aligned}
a &= \frac{Y}{\sqrt{r}}\left(1 + \frac{1-|Y|^2/2}{r} + \cdots\right) + Cre^{-r}\left(1 + \frac{\gamma}{r} + \cdots\right),\\
g &= \frac{1}{2}\log 2r - \frac{|Y|^2}{2r^2}(1+\cdots) + P\sqrt{r}e^{-r}\left(1 + \frac{\alpha}{r} + \cdots\right),\\
\Phi &= \Phi_0 + \frac{r}{2} - \log\frac{r}{4} + \frac{5|Y|^2}{16r^2}(1+\cdots) - P'\sqrt{r}e^{-r}\left(1 + \frac{\beta}{r} + \cdots\right),
\end{aligned} \tag{3.3}$$

where dots stand for corrections in $1/r$. Φ_0 is a free parameter determining the dilaton (coupling) at a given r (scale). We interpret the complex parameters Y (the non-normalizable

[4]We might expect the relative scaling dimension of the two sources to be apparent from the r dependence of the solution in the UV (for example, naively, one scaling as r the other as r^3) but in this case there does not seem to be a straightforward interpretation.

solution) as the mass deformation, and C (the normalizable solution) as the complex conjugate of the condensate. The functions P, P', α, β and γ are determined by the equations of motion and we find

$$P = P' = k\,\mathrm{Re}(\bar{C}Y)\,, \qquad \alpha = 2 + \frac{1}{2k}\,, \qquad \beta = 1 + \frac{1}{2k}\,, \qquad \gamma = \frac{1 + (4k+3)|Y|^2}{2}\,, \tag{3.4}$$

where k is a free parameter.[5] Finally we would expect two other free parameters in the solution of these equations which are encoded in the freedom to shift and scale the r coordinate

$$r \longrightarrow \mu(r + r_*)\,. \tag{3.5}$$

Clearly there are many more free parameters than we expect in the field theory which is uniquely determined by the UV value of the mass and the coupling. However from the analysis of the IR solutions [17] we see that there is a two parameter family of regular solutions given by

$$\begin{aligned} a &= e^{i\chi'}(1 - br^2 + \cdots)\,, \qquad e^g = r - \left(b^2 + \frac{1}{36}\right) r^3 + \cdots\,, \\ \Phi &= \phi(0) + \left(b^2 + \frac{1}{12}\right) r^2 + \cdots\,. \end{aligned} \tag{3.6}$$

Restricting to these solutions, the eight real parameters in the UV are reduced to three. $\phi(0)$ matches to g^2_{YM}, and b to the gaugino mass term. The regular IR solution has only a single phase in the field a whilst the UV solution has two, one on Y and one on C. Regularity in the IR therefore forces the phase of the condensate and the conjugate of the mass term to be equal. This is precisely the condition for the minimum of the field theory potential in (1.3) and is our first hint that the gravity solution correctly encodes the field theory.

The full solutions can be found by numerically integrating the IR solutions to the UV and solving for the UV parameters as a function of b and $\phi(0)$. In the range $b \in [0, 1/6]$, the solutions have a regular behaviour [17] and we can interpret them as mass deformations of the MN solution. At the supersymmetric point in the IR $b = 1/6$ whilst in the UV $\mu = 1, r_* = -1/2$, $C = 2/\sqrt{e}$ and $Y = 0$ (Φ_0 or $\phi(0)$ is a free choice). In fact to determine the vacuum energy of the regular configurations we shall only need the UV asymptotic forms of the solutions. There is a subtlety though; we will need to know the value of the parameter k when we break supersymmetry. At the supersymmetric point k is undetermined because $Y = 0$, however, its value can be found as the limiting value of k as $b \to 1/6$. This requires the numerical integration procedure described above. In figure 1 we show $k(b)$ close to $b = 1/6$. The numerical procedure necessarily runs into trouble at $b = 1/6$ but the limiting value can be read off to be approximately -390. All we will need is that k is a constant plus $\mathcal{O}(b) \sim \mathcal{O}(m_\lambda)$. Note that k is a $U(1)_R$ invariant quantity and hence has the same value independent of the phases on Y and C.

[5] Note that our solution differs a little from that in [17]. They present as the solution the limit of our equations where $k \to \infty$ with $k\,\mathrm{Re}(\bar{C}Y) \to$ constant. The parameter P then becomes free. In fact the result that P is proportional to $\mathrm{Re}(\bar{C}Y)$ is crucial to our analysis of the vacuum energy below.

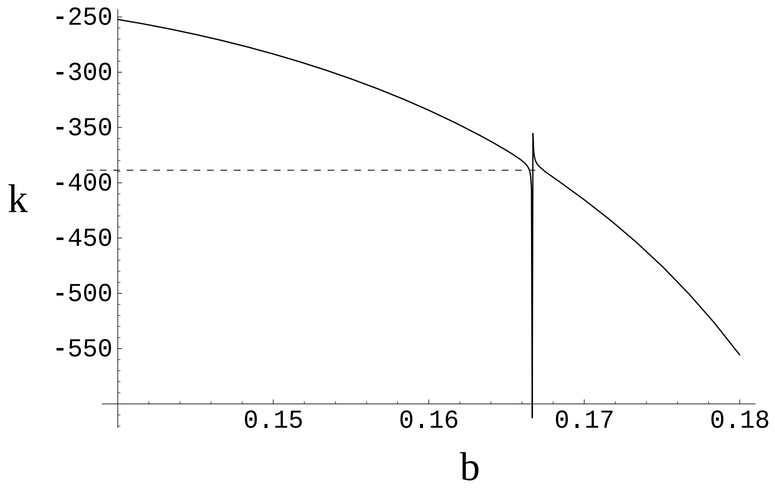

Figure 1: plot of k vs b around the supersymmetric point $b = 1/6$.

Finally we recall that these solutions can be lifted to ten dimensions, in the same way as the Maldacena-Nunez solution, giving

$$\begin{aligned} ds^2_{10\ \mathrm{str}} &= e^{\Phi}\left[dt^2 + dx^2 + \frac{1}{4}dr^2 + \frac{1}{4}e^{2g}(d\theta^{+}\sin^2\theta d\phi^2) + \frac{1}{4}\sum_a (w^a - A^a)^2\right], \\ e^{2\Phi - 2\Phi_0} &= e^{-g}\sinh r, \\ C_2^{RR} &= N\left[\frac{1}{4}(w^1 - A^1)\wedge(w^2 - A^2)\wedge(w^3 - A^3) - \frac{1}{4}\sum_a F^a \wedge (w^a - A^a)\right], \quad (3.7) \end{aligned}$$

where w_i is a set of 1-forms describing a three-sphere. It is interesting to notice that the deep IR form of the metric is exactly the same for all solutions, supersymmetric and not. We interpret this as meaning that the theories all possess a mass gap and the deep IR is therefore the same for all these theories. The study of the full solution would reveal the differences in the spectrum and the dynamics due to the inclusion of a mass term.

4. Vacuum energy

We have seen that the supergravity equation of motions give rise to a set of solutions in the UV differing only in the phase of the gaugino in the condensate, C. We will therefore be interested in comparing solutions of the second order equations with fixed gaugino mass, Y, and varying phase condensate C. In this section we will compute the relative vacuum energy of these space-times to determine the true vacuum. The expected form in the field theory is reproduced by the UV behaviour of the solution. For the regular solution, which we identified above with the true vacuum, we can therefore reproduce the field theory true vacuum energy's dependence on the breaking mass term. For the metastable vacua the IR solutions are not regular and the prescription to deal with the IR is less clear.

Since the UV contribution matches the field theory expectation, at leading order in the mass term we expect that corrections due to the unknown IR behaviour are subleading for small mass.

The value of the euclidean action for this family of solutions can be found in [17] and we briefly review the computation. Since the 7-dimensional solution has a non-trivial dependence only on the radial and the 2-sphere coordinates, in the following computation we can neglect the contribution of the 3 flat spatial directions (they will only provide a divergent multiplicative constant factor). We then reduce ourselves to the following euclidean 4-dimensional action in the Einstein frame

$$I = \frac{1}{4\pi}\int_{\mathcal{M}} d^4y\sqrt{g}\left(-\frac{1}{4}R + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{8}e^{2\Phi}F^a_{\mu\nu}F^{a\ \mu\nu} - \frac{1}{4}e^{-2\Phi}\right) - \\ - \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^3y\sqrt{h}K\,. \tag{4.1}$$

The surface term in the expression above is the contribution from the extrinsic curvature $K = \sqrt{g^{rr}}h^{ab}\partial_r h_{ab}, = e^{-2g-4\Phi}\frac{\partial}{\partial r}(e^{2g}e^{3\Phi})$, where h_{ab} is the 3-dimensional surface metric, $h = e^{2\Phi}(d\tau^2 + e^{2g}(d\theta^2 + \sin^2\theta d\phi^2))$. Explicitly the boundary integral reads

$$I_{bd} = -\frac{1}{2}\lim_{r\to\infty} e^{-\Phi}\partial_r\ \left(e^{2g+3\Phi}\right). \tag{4.2}$$

Using the equations of motion the volume term reduces to a surface integral since for the regular solutions the $r \to 0$ limit of the integration vanishes. The analysis is thus appropriate only to the regular non-supersymmetric solution which as we described above aligns the phases of the mass and the condensate in the UV. In fact the result we find below reproduces the field theory result even for the metastable vacua, so most probably, at leading order in Y, the computation is valid for all the solutions.

$$I_{\text{bulk}} = \frac{1}{8\pi}\int d^4x\ \partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\mu\Phi) = \lim_{r\to\infty}\frac{1}{2}e^{2g}e^{2\Phi}\partial_r\Phi\,. \tag{4.3}$$

The action is therefore given by

$$I \sim \lim_{r\to\infty}\partial_r\left(e^{2g}e^{2\Phi}\right). \tag{4.4}$$

Notice that it does not explicitly depend on the field a. Let us use this result to compute the vacuum energy in the softly broken theory. Substituting in the solution (3.3) we find for the leading term

$$I \sim \lim_{r\to\infty}\partial_r\left(2\sqrt{r}e^{(r+2\Phi_0)}\right). \tag{4.5}$$

This is a divergent piece common to all the solutions. This piece will cancel when we compare the energies of any two solutions. When we make a comparison between two space-times we must be careful to make sure that the metric on the boundary is the same for each of the two space-times.

We shall use the MN zero gaugino mass spacetime as our reference geometry. For that metric, using the freedom to shift r and Φ_0 we find

$$I_{\text{BPS}} \sim \lim_{r\to\infty}\partial_r\left(2\frac{e^{(r+r_*+2\Phi_0+2\Phi_*)}}{\sqrt{r+r_*}}\right). \tag{4.6}$$

In particular, as in [17] we must choose the constants r_* and Φ_* so the dilaton and the S^2 metric are the same in both cases. In the field theory this corresponds to equating the gauge coupling at the UV scale. We note that the coefficient of the S^2 metric is

$$e^{2g+2\Phi} \simeq 2\sqrt{r}e^{(r+2\Phi_0)}(1+\cdots) + 2e^{2\Phi_0}(2\alpha P - 2\beta P)(1+\cdots)\,, \tag{4.7}$$

where dots stand for polynomials terms in $1/r$. We also note that the polynomial corrections to the leading $\sqrt{r}e^r$ behaviour only depend on Y and not on C.

We first compute the vacuum energy at linear order in the mass. Keeping only linear terms in Y, the matching gives

$$\begin{aligned}\sqrt{r+r_*}e^{(r+r_*+2\Phi_0+2\Phi_*)} &= \sqrt{r}e^{(r+2\Phi_0)} + e^{2\Phi_0}(2\alpha P - 2\beta P)\,,\\ \sqrt{2(r+r_*)} &= \sqrt{2r} + \sqrt{2}Pre^{-r}(1+\cdots)\,.\end{aligned} \tag{4.8}$$

The first equation can be used to fix Φ_*. The second one then fixes $r_* = 2Pr^{3/2}e^{-r}+\cdots$. The energy difference is therefore[6]

$$\Delta I = e^{2\Phi_0}2k\,\mathrm{Re}\left(\bar{C}Y\right). \tag{4.9}$$

In the supersymmetric limit where Y, the gaugino mass, is zero the solutions with different phases on the (complex conjugate of the) condensate C are degenerate. When a gaugino mass is introduced the energy of the vacua to leading order in Y (or m_λ) is given by

$$E \sim \mathrm{Re}(\bar{C}Y) \sim \mathrm{Re}\left(m_\lambda \Lambda^3\right), \tag{4.10}$$

reproducing the field theory result (1.3). Note it was crucial here that k = constant $+\mathcal{O}(m_\lambda)$ as we showed numerically above.

We can repeat the above calculation to higher orders in Y using the asymptotics (3.3) and (4.7). The result is that divergent terms of the form $\sim |Y|^2 e^r/r^{5/2} + \cdots$ appear in the vacuum energy. All these terms depend on the mass Y but not on the condensate C and reflect the fact that the vacuum energy in the softly broken theory is infinite (the leading vacuum graph is a fermion loop with two mass insertions). They cancel when computing difference in the energy for different vacua. The final result for the vacuum energy is still given by eq. (4.9). As mentioned before, Y, C and k are complicated functions of the mass parameter, which can be found by matching the UV and IR behaviours of the metric. Formula (4.9) therefore encodes all higher order corrections in the mass parameter.

5. Conclusions

We have studied softly broken $\mathcal{N}=1$ theories by deforming the Maldacena-Nunez solution. We have computed the vacuum energy and verified that the $\mathcal{N}=1$ degeneracy of vacua is lifted according to expectations.

[6]The formula $I = 2e^{2\phi_0}P$ was obtained in [17]. The authors of [17] were, however, interested in solutions with $Y = 0$, interpreted as non-supersymmetric finite energy excitations of the MN solution, where P becomes an extra parameter. In our case, P is fixed in term of Y by eq. (3.4)

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Information about condensates and vacuum energy are encoded in the subleading UV behaviour of the solution, once parameters and asymptotics are fixed by boundary conditions and regularity in the IR. It would be interesting to study other features of the softly broken theory encoded in the full solution or in the IR behaviour, for example, to compute the glue-ball spectrum in the Maldacena-Nunez solution and in its deformations. Another interesting quantity is the ratio of k-strings in this model. It was noticed in [23] that in the MN solution the ratio of tensions follows the sine formula

$$\frac{T_k}{T_{k'}} = \frac{\sin k\pi/N}{\sin k'\pi/N} \tag{5.1}$$

found in $\mathcal{N} = 2$ SYM [24], MQCD [25] and somewhat supported by recent lattice computations [26]. It appears that, since the string tension is fixed by the IR behaviour, the string ratio in the softly broken theory is the same as in the Maldacena-Nunez solution, that is it follows formula (5.1).

The sine formula, or mild modifications of it, are quite commonly realized in stringy inspired models of YM, even if it is known that QFT provides some counterexamples to the universality of such a formula [27].

Finally, we should discuss the issue of stability. The solutions we considered could be unstable, since supersymmetry is not protecting them anymore. However, since the $\mathcal{N} = 1$ gauge theory has a mass gap, and the MN solution is expected to have a discrete tower of normalizable fluctuations, we could expect that, at least for small deformations, stability is preserved. A more detailed analysis is nevertheless necessary to determine the absence of tachyons in the background.

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